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Arbitrary Accuracy Integration Scheme for the Subsonic Doublet Lattice Method

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Nomenclature

D_1, D_2 = planar and nonplanar parts of the incremental oscillatory downwash factor
e = box semispan
k_r = reduced frequency based on box semichord or mean wing semichord
M = Mach number
$P^n(x)$ = polynomial of degree n in x
Δx = box chord
y, z = coordinates of collocation point in sending panel coordinate system
η = spanwise coordinate of sending point

Introduction

THE subsonic doublet lattice method,[1,2] used in the calculation of unsteady air loads on an aircraft, can be regarded as an extension of the vortex lattice method, which is used for steady load calculation. The downwash factors are calculated as the sum of the steady component, identical to that of the vortex lattice method, and an unsteady component. Whereas the steady component is exact, the unsteady component is approximated. The approximation in the unsteady component places a restriction on the doublet lattice method in terms of box aspect ratio, which does not apply to the vortex lattice method. The incremental oscillatory downwash factor is divided into a planar and a nonplanar part.[2] Each part is expressed as an integral over the length of the doublet line, of which the integrand is the planar or nonplanar kernel numerator divided by the square or fourth power, respectively, of the radial distance between the sending and receiving points. These integrals cannot be evaluated analytically, and the usual way of evaluating them is to make polynomial approximations to the numerators of the integrands and integrating the resulting expressions analytically. To obtain accurate results, the box aspect ratio must be limited. Reference 2, which describes the use of parabolic approximations, suggests using box aspect ratios not much greater than unity, whereas Ref. 3 suggests using box aspect ratios of less than three, also for a parabolic approximation.

Rodden et al.[4,5] described quartic approximations to the kernel numerators and suggested that the higher degree approximation may allow the use of aspect ratios of up to 10. To quantify the integration error in these production methods, it would be useful to have available a method that will converge to the correct result in an orderly manner as computational effort is increased. Increasing the degree of the approximating polynomials does not have the desired effect. The excursions between matched points get larger and the results diverge. The approach presented here is to make a natural cubic spline approximation to the kernel numerators, followed by analytical integration over each interval. General integration formulas, necessitated by the nonsymmetric integration intervals, are given. Results are presented that indicate that the method converges without difficulty.

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General Integration Formulas

The incremental (unsteady) downwash factor is divided into a planar and a nonplanar part as was done in Ref. 2, but polynomials of arbitrary degree are substituted for the parabolas

$$D_1 = \frac{\Delta x}{8\pi}\int_{\eta_1}^{\eta_2} \frac{P_1^n(\eta)}{(y-\eta)^2+z^2}\,d\eta \qquad (1)$$

$$D_2 = \frac{\Delta x}{8\pi}\int_{\eta_1}^{\eta_2} \frac{P_2^n(\eta)}{[(y-\eta)^2+z^2]^2}\,d\eta \qquad (2)$$

The numerators are divided by the denominators to obtain

$$D_1 = \frac{\Delta x}{8\pi}\int_{\eta_1}^{\eta_2} \left[P^{n-2}(\eta) + \frac{a_1\eta + a_0}{(y-\eta)^2+z^2}\right] d\eta \qquad (3)$$

$$D_2 = \frac{\Delta x}{8\pi}\int_{\eta_1}^{\eta_2} \left[P^{n-4}(\eta) + \frac{b_3\eta^3 + b_2\eta^2 + b_1\eta + b_0}{[(y-\eta)^2+z^2]^2}\right] d\eta \qquad (4)$$

The polynomials are simple to integrate, whereas the method of partial fractions can be used to integrate the fractions. The fraction in the nonplanar integrand is expressed in terms of its partial fractions as

$$\frac{a+ib}{\eta-(y+iz)} + \frac{a-ib}{\eta-(y-iz)} + \frac{c+id}{[\eta-(y+iz)]^2} + \frac{c-id}{[\eta-(y-iz)]^2} \qquad (5)$$

where

$$a = b_3/2 \qquad (6)$$

$$b = -[b_0 + yb_1 + (y^2+z^2)b_2 + y(y^2+3z^2)b_3]/4z^3 \qquad (7)$$

$$c = -[b_0 + yb_1 + (y^2-z^2)b_2 + y(y^2-3z^2)b_3]/4z^2 \qquad (8)$$

$$d = -[b_1 + 2yb_2 + (3y^2-z^2)b_3]/4z \qquad (9)$$

The first two partial fractions integrate to logarithmic functions, whereas the last two integrate to simple reciprocals. If $z = 0$, the result for the planar part is given by

$$D_1 = P^{n-1}(\eta_2) - P^{n-1}(\eta_1) + a_1 \ln\left|\frac{y-\eta_2}{y-\eta_1}\right| + (a_1 y + a_0)\left(\frac{1}{y-\eta_2} - \frac{1}{y-\eta_1}\right) \qquad (10)$$

If $z \neq 0$, the planar and nonplanar parts are given by

$$D_1 = P^{n-1}(\eta_2) - P^{n-1}(\eta_1) + \frac{a_1}{2}\ln\frac{(y-\eta_2)^2+z^2}{(y-\eta_1)^2+z^2} + \frac{a_1 y + a_0}{2z}\arg[f + iz(\eta_2-\eta_1)] \qquad (11)$$

$$D_2 = P^{n-3}(\eta_2) - P^{n-3}(\eta_1) + a\,\ell n \frac{(y-\eta_2)^2 + z^2}{(y-\eta_1)^2 + z^2}$$
$$- 2b \arg[f + iz(\eta_2 - \eta_1)]$$
$$+ (\eta_2 - \eta_1)\frac{2cg - 2\,dz(\eta_2 + \eta_1 - 2y)}{g^2 + z^2(\eta_2 + \eta_1 - 2y)^2} \quad (12)$$

where

$$f = (y - \eta_2)(y - \eta_1) + z^2 \quad (13)$$

$$g = (y - \eta_2)(y - \eta_1) - z^2 \quad (14)$$

The Fortran ATAN2 function can be used to evaluate the arg function in the preceding expressions. By substituting $\eta_2 = +e$ and $\eta_1 = -e$, expressions for integrating over the entire doublet line are obtained.

Spline Approximation

A natural cubic spline through a number of points is a set of cubic polynomials defined over each interval such that the first and second derivatives are continuous. At the endpoints, the second derivative is set equal to zero. It is a simple matter to make a spline fit instead of a polynomial fit to the numerator, and integrate the resulting set of cubic polynomials. One complication is that it is desirable to have a matched point at the center of the doublet line in the case of a collocation point in the same strip. However, the integration formulas are singular for a collocation point at the endpoint of the interval. To avoid this problem, the two central cubic polynomials are replaced by a single quartic polynomial that matches the central three function values and joins the adjacent cubics with the same conditions as those between cubics, i.e., continuous first and second derivatives. This method was applied to the case of a 70-deg swept box of aspect ratio 10, $M = 0.8$, and $k_r = 0.25$. Spline approximations to the planar kernel numerator using 5, 9, and 17 points, respectively, are shown in Fig. 1. The spline fit converges to the original function without difficulty.

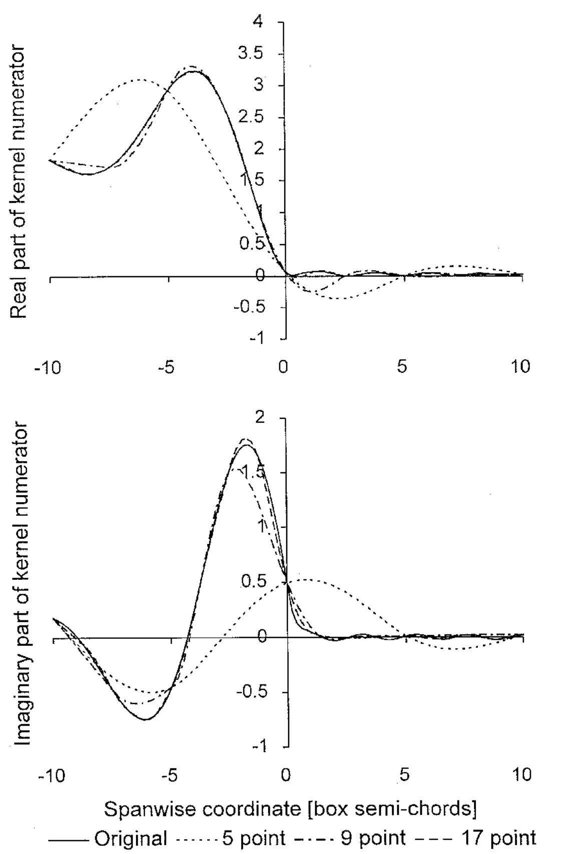

Fig. 1 Spline approximations to the planar kernel numerator.

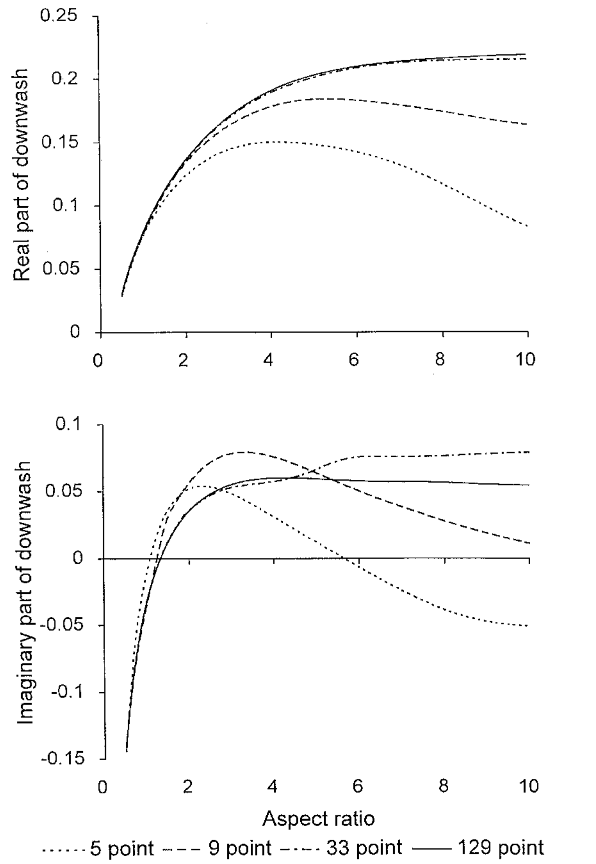

Fig. 2 Unsteady downwash calculated using spline approximations.

Integrated Results

The fit quality considered in the previous text does not give an intuitively correct impression of the accuracy of the downwash factor because of the weighting introduced by the denominator in the integrands and the principal value of the integral being taken in the planar case. The convergence behavior of the present method is illustrated by plotting the planar downwash factor against box aspect ratio for a 70-deg swept box, $M = 0.8$ and $k_r = 0.25$. In Fig. 2, results obtained using 5-, 9-, 33-, and 129-point spline approximations are shown. Although many points are required for convergence in this extreme case, convergence is achieved without difficulty.

Conclusions

The present study has shown that results of high accuracy can be obtained at the cost of computational effort by using a spline fit to the kernel numerators and integrating over each interval analytically. This procedure can be used to quantify the integration error in production methods. It should however be noted that eliminating the integration error for a particular paneling scheme does not yield the fully converged answer for the problem, nor is it necesary to eliminate the integration error to determine the fully converged result.

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Convergence of the Subsonic Doublet Lattice Method

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Nomenclature

C_l = lift coefficient based on wing area
h = plunge amplitude
k = reduced frequency based on wing semispan
k_r = reduced frequency based on mean wing semichord
M = Mach number
n = number of boxes in aerodynamic model
n_c = number of chordwise boxes on wing
s = wing semispan

Introduction

THE subsonic doublet lattice method[1,2] (DLM) is commonly used for the calculation of unsteady air loads on aircraft. This method can be regarded as an extension of the vortex lattice method (VLM), which is used for steady load calculation. The downwash factors in the DLM are calculated as the sum of a steady component, identical to that of the VLM, and an unsteady component. Whereas the steady component is exact, the unsteady component is approximated. Both the VLM and DLM suffer from a discretization error, the error introduced by dividing a lifting surface into finite panels. The DLM is much more sensitive to chordwise paneling than the VLM because of the oscillatory nature of the downwash of a lifting surface element in unsteady flow. In addition, the DLM suffers from an integration error, the error resulting from inaccuracies in the calculation of the unsteady component of the downwash factors. To obtain acceptably accurate results from the DLM, application guidelines must be adhered to.

Historically, the application guidelines separated the discretization and integration errors.[2,3] A minimum number of chordwise boxes per wavelength is specified to limit the discretization error, whereas a maximum box aspect ratio is specified to limit the integration error. Reference 2, which describes the use of parabolic approximations to the kernel numerators, suggests using box aspect ratios not much greater than unity and at least 25 boxes per wavelength (at the wing root). Reference 3 suggests using box aspect ratios less than three and at least 12.5 boxes per wavelength, also for a parabolic approximation.

For most aeroelastic applications, the combination of these two conditions dictate a fine spanwise paneling. Therefore, it was not necessary to address the spanwise paneling requirement explicitly. Rodden et al.[4,5] described quartic approximations to the kernel numerators and suggested that the higher degree approximation may allow the use of aspect ratios of up to 10. If such high box aspect ratios are used, attention will have to be given to convergence with respect to spanwise paneling. A minimum number of 50 chordwise boxes per wavelength (at mid-span) was suggested in Ref. 5, which is significantly more than what had previously been suggested for the parabolic approximation.

The guidelines for the number of boxes per wavelength were arrived at by refining the chordwise paneling and noting when the results became stationary to within the desired limit. The higher box aspect ratio limit suggested in Ref. 5 was arrived at by continuing the refinement of the chordwise paneling and noting that the results did not deviate from the stationary result. The discretization and integration errors were not separated in these studies. In the present study, the two sources of error are separated using a much more accurate approximation to the kernel numerators.[6]

Convergence with Respect to Chordwise Paneling

Results for a two-dimensional case are presented first to get an indication of the expected convergence behavior of the DLM in the absence of any spanwise integration error. The test case is that of an airfoil oscillating in pitch, for which Von Kármán and Sears[7] gave an analytical closed-form solution. An incompressible two-dimensional DLM was used to calculate the ratio of the unsteady moment to the steady moment at $k_r = 2$. Only the contribution of the pitching motion to the boundary condition was used, as was done in Ref. 7. Results for 16–1024 chordwise panels are plotted against $1/n$ in Fig. 1. The real and imaginary parts of the analytical result are indicated by short horizontal lines drawn onto the vertical axis. It is seen that the DLM results converge to the analytical result along approximately straight lines. The DLM results were extrapolated to an infinite number of panels ($1/n = 0$) from the last two points of each line and compared to the analytical result. The relative error, i.e., the magnitude of the difference divided by the magnitude of the analytical result, is 0.003%.

The AGARD wing and horizontal tail in plunge at $M = 0.8$ and $k_r = 1.2$ is used to illustrate the convergence behavior, with respect to chordwise paneling, of the DLM for different levels of integration error. The wing and tail are divided into eight strips at 1/6, 1/3, 1/2, 2/3, 5/6, 0.9, and 0.96 fractions of

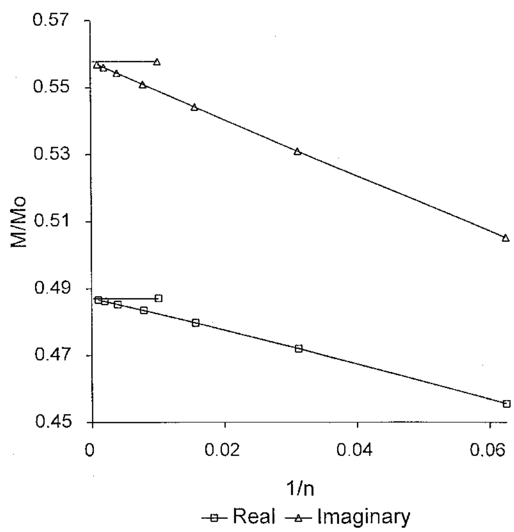

Fig. 1 Moment on an airfoil oscillating in pitch.

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